Pre-Calculus—Lesson 1 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

Find all the real solutions, if any, of each equation.

1. $|3x + 1| = 5$

2. $\frac{4x-5}{3-7x} = 4$

Solve each inequality.

3. $-4 < \frac{2x-2}{3} < 6$

4. $|1-2x| < \frac{1}{3}$

Find an equation of the line having the given characteristics.

5. X-intercept = 2; passing through (4, -5).

6. Perpendicular to the line $3x - y + 4 = 0$; passing through (-2, 2).

Graph the line and label the x- and y-intercepts.

7. $\frac{-3x}{4} + \frac{y}{2} = 0$

Find the radius of the circle.

8. $x^2 + y^2 + 4x - 4y - 1 = 0$

Pre-Calculus—Lesson 2 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

In problems 1 through 4,

(a) Find the x- and y-intercepts of each polynomial function;

(b) Determine whether the graph of f touches or crosses the x-axis at each x-intercept.

1. $f(x) = x\,(x - 2)\,(x - 4)$

2. $F(x) = (x - 2)\,(x + 4)^2$

3. $F(x) = x^3 + 4x$

4. $F(x) = (x - 4)\,(x + 2)^2\,(x - 2)$

Use synthetic division to determine the quotient in problems 5 through 6.

5. $F(x) = 2x^3 + 8x^2 - 5x + 5; \quad G(x) = x - 2$

6. $f(x) = x^4 - x^2 + 3x; \quad g(x) = x + 1$

7. Find the value of $f(x) = -16\,x^3 + 18\,x^2 - x + 2$ at $x = -2$.

In problems 8 and 9, use Descartes' Rule of Sign to determine how many positive and negative zeros each polynomial function may have. Do not attempt to find the zeros.

8. $f(x) = -6x^5 + x^4 + 5x^3 + x + 1$

9. List all potential rational zeros of $f(x) = -6x^5 + x^4 + 2x^3 - x + 1$

Pre-Calculus—Lesson 3 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

In problems 1 and 2, the function given is one-to-one. Find the inverse of each function. Find the domain and range of f and f^{-1}.

1. $f(x) = \dfrac{2-x}{3+x}$

2. $f(x) = x^{\frac{1}{3}} + 1$

In problems 3 and 4, write each expression as a single logarithm.

3. $-2\log_3 \dfrac{1}{x} + \dfrac{1}{3}\log_3 \sqrt{x}$

4. $\dfrac{1}{2}\ln(x^2+1) - (4)\ln\left[\dfrac{1}{2}\right] - \dfrac{1}{2}[\ln(x-4) + \ln x]$

In problem 5, find y as a function of x. The constant C is a positive number.

5. $\ln(y - 3) = \ln 2x^2 + \ln C$

In problems 6 to 9, solve each equation.

6. $8^{6+3x} = 4$

7. $\log_x 64 = -3$

8. $2^{x+1}\, 8^{-x} = 4$

9. $\log_{10}(7x - 12) = 2\log_{10} x$

10. Convert 210° to radians. Express your answer as a multiple of pi.

Convert the angle in problem 11 to degrees.

11. Convert $\dfrac{-3\pi}{2}$ to degrees.

Pre-Calculus—Part 1 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite them on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with attached work (not the exam itself) to Keystone National for grading. Good Luck!

1. Find all the real solutions, if any, of $(6 - 3x) - 2(1 + x) = 6x$

2. $\frac{-2}{1-3x} < -1$

3. Find an equation of the line passing through (3,-4) and (2,1).

4. Find the center and radius of $2x^2 + 2y^2 - 4x = 0$

5. Find the x- and y-intercepts of $y = 5x$ and test for symmetry.

6. Find for $\frac{x^3}{x^2 - 4}$:

   (a) $f(-x)$ (b) $-f(x)$

   (c) $f(x+2)$ (d) $f(x-2)$

7. Determine algebraically whether $f(x) = 1 + x + x^2$ is even, odd or neither.

8. Find the domain of $f(x) = \frac{1}{x^2 - 3x - 4}$.

9. $f(x) = 4 - x$ and $g(x) = 1 + x^2$; find:

   (a) $(f \circ g)(2)$ (b) $(g \circ f)(-2)$

   (c) $(f \circ f)(4)$ (d) $(g \circ g)(-1)$.

Pre-Calculus—Lesson 4 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

In problems 1 to 4, establish each identity.

1. $\sin A \csc A - \sin^2 A = \cos^2 A$

2. $(1 - \cos^2 A)(1 + \cot^2 A) = 1$

3. $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\csc A$

4. $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

In problems 5 and 6, find the exact value of each expression.

5. $\sin 70° \cos 40° - \cos 70° \sin 40°$

6. $\cos^{-1} 0$

7. Solve $\cos 2A = 0$ on the interval $0 \leq A \leq 2\pi$

In problems 8 and 9, solve each right triangle.

8. $A = 35°$ and side $a = 5$.

9. Side $a = 3$ and side $b = 1$.

Pre-Calculus—Lesson 5 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

In problem 1, the rectangular coordinates of a point are given. Find two pairs of polar coordinates, one with $r > 0$ and one with $r < 0$. Express the angles in radians.

1. (1, -1)

2. In $3r = \sin A$, r and A represent polar coordinates. Write each polar equation as an equation in rectangular coordinates (x,y).

In problem 3, express each complex number in polar form. Express each angle in degrees.

3. $\sqrt{3}+i$

4. Write $3(\cos 60° + i \sin 60°)$ in the standard form a + bi.

5. Find zw and $\frac{z}{w}$. Leave your answer in polar form

   $z = \cos 205° + i \sin 205°$

   $w = \cos 85° + i \sin 85°$

6. Write $\left[2\left(\cos \frac{5\pi}{16} + i \sin \frac{5\pi}{16}\right)\right]^4$ in the standard form a + bi.

Pre-Calculus—Lesson 6 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite your answers on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with work attached (not the exam itself) to Keystone National for grading. Good Luck!

Be sure to show your work so that you may get some partial credit.

Evaluate the following terms.

1. 6!

2. P(7,3)

3. Write down the first five terms of $\frac{e^n}{n}$

Determine whether the given sequence is arithmetic, geometric or neither. If it is arithmetic, find the common difference and the sum of the first n terms. If it is geometric, find the common ratio and the sum of the first n terms.

4. 3^{2n}

5. 1, -3, -7, -11, ...

6. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

Find the sum of each sequence.

7. $\sum_{k=1}^{9} (-2K + 8)$

8. $\sum_{k=1}^{10} (-2)^k$

Pre-Calculus—Part 2 Examination

Please read these instructions before you begin to answer the questions that follow.

Write your answers and your work on this examination. When you are satisfied that your answers are correct, rewrite them on the Answer Sheet found right after the exam. Attach separate paper showing your work. Then send the Answer Sheet with attached work (not the exam itself) to Keystone National for grading. Good Luck!

Establish each identity.

1. $4\sin^2 A + 2\cos^2 A = 4 - 2\cos^2 A$

2. $1 - \dfrac{\cos^2 A}{1+\sin A} = \sin A$

3. Find the exact value of $\tan 105^\circ$.

4. Find the exact value of $\sin^{-1}\left[-\dfrac{1}{2}\right]$

5. Solve: $\sin 3A = 1$

6. Find the remaining angles and side if they exist. If no triangle exists, say "No triangle."

   $a = 4;\ b = 1;\ \gamma = 100^0$

7. Find the area of the triangle: $a = 2;\ b = 1;\ \gamma = 100^0$

8. (-5,12) are the rectangular coordinates of a point. Find two pairs of polar coordinates (r,A) for each point, one with r>0, and one with r<0. Express A in radians.

9. Write $A = \dfrac{\pi}{4}$ as an equation in rectangular coordinates, (x,y).

18. Solve using matrices. If the system has no solution, say it is inconsistent.

$$x + 2y - z = 2$$
$$2x - 2y + z = -1$$
$$6x + 4y + 3z = 5$$

19. Find the determinant:

$$\begin{vmatrix} 2 & 3 & 10 \\ 0 & 1 & 5 \\ -1 & 2 & 3 \end{vmatrix}$$

20. Use Cramer's Rule, if applicable, to solve the system.

$$3x - 4y - 12 = 0$$
$$5x + 2y + 6 = 0$$